HEAT TRANSFER IN INTENSIVE NONSTATIONARY PROCESSES

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A theory is exposed of intensive nonstationary heat-conduction processes, based on energy transfer mechanisms by means of carriers, emitted and absorbed by particles of the material. Results are provided of solving the integrodifferential transport equation by a difference method.

The heat transport differential equations usually applied do not make it possible to account for the effect of the spatial scale of the body on its transport characteristics, and ultimately on the temperature field. This effect is substantial for intensive nonstationary processes, when the heat carrier mean free path is commensurate with the spatial scale of the body or that of the temperature field.

Account of this effect, as well as of the finite propagation velocity of temperature perturbations, can be provided on the basis of the energy transport mechanism by means of carriers, emitted and absorbed by particles of the material [1].

Consider the energy state of some particle α belonging to a solid. The heat balance equation

$$\frac{\partial E_\alpha}{\partial \tau} = \sum_{\gamma=1}^{\Gamma} \sigma_{\alpha\gamma} J_{\gamma\alpha} - q_\alpha + w_\alpha \qquad (1)$$

is valid for it. In this equation the product $\sigma_{\alpha\gamma}J_{\gamma\alpha}$ is the energy acquired by particle α from particle γ per unit time. The summation over Γ particles provides the total energy acquired by particle α from the other particles of the body per unit time. The energy lost by particle α per unit time due to carrier emission is represented by the term q_α. The last term in the right hand side of the equation is the power of external and internal energy sources.

Equation (1), valid for any particle of the body, contains three unknown functions, E_α, q_α, and $J_{\gamma\alpha}$. Therefore, to solve it one needs two additional relations, relating the functions q_α, $J_{\gamma\alpha}$ and E_α. The functional dependence between the energy q_γ, emitted by particle γ per unit time, and the specific carrier energy flux $J_\gamma(\eta)$, created by it through a spherical surface of radius η is determined by the structure of the body and the shape of the carrier. We place the origin of spherical coordinates at the center of particle γ, and partition the volume surrounding it by surfaces η = const into elementary volumes $dV = 4\pi\eta^2 d\eta$. The energy of carriers emitted by particles γ, absorbed in the volume dV of a homogeneous body, is $dQ_\gamma = J(\eta)4\pi\eta^2 d\eta F$, where F is the total effective cross section of particle absorption per unit volume.

The quantity $J(\eta)4\pi\eta^2$ is the energy flux $Q_\gamma(\eta)$, transported through a spherical surface of radius η by the carriers emitted by particle γ. Bearing in mind that carriers which were emitted by particle γ at the moment of time τ are incident on particle removed from particle γ at moment $\tau-\eta/v$, as well as that in the absence of absorption by carriers $Q_\gamma(\eta, \tau) = q_\gamma(0, \tau-\eta/v)$, following integration over dQ_γ we obtain

$$Q_\gamma(\eta, \tau) = q\left(0, \tau - \frac{\eta}{v}\right) G(\eta), \; G(\eta) = \exp\left(-\int_0^\eta F d\eta\right). \qquad (2)$$

If the energy carriers are photons, v is the speed of light.

For amorphous bodies, whose particles are disorderly arranged with equal probability at any portion of the volume, $F = \sum_{s=1}^{S} \sigma_s n_s$. In bodies of an ordered structure a chain of other

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sites is located at equal intervals at each line passing through two sites of the crystal lattice. Placing the particle centers at the sites of this chain, each particle can directly exchange carriers with two neighbors only, since the remaining particles are in the shadow. This fact decreases the probability of carrier absorption by particles, and is equivalent to a decrease in the particle absorption cross section. For these bodies $F = \mu\sum_{s=1}^{S}\sigma_s n_s$, where the particle overlap coefficient is $\mu > 1$. For anisotropic bodies the function F has a more complicated shape [1]. From the energy balance equation it follows for particles of an isotropic unbounded body in the equilibrium state that the function G satisfies the condition

$$\int_{\eta=0}^{\infty} FG d\eta = 1. \tag{3}$$

The dependence q = q(E) is determined by the physical properties of the body particles, and is unrelated to the configuration and state of the latter. To establish it consider an amorphous planar plate in a stationary nonequilibrium state, in which energy transfer is realized by one of the carrier shapes. We direct the x-axis normally to the plate boundary, and the ζ-axis - along the boundary. The temperature field in the plate is expressed by the dependence t = t(x). It follows from experimental data that the thermal conductivity coefficient λ of amorphous bodies is directly proportional to the specific heat capacity. In this connection it is noted for the plate considered that $\lambda = a\rho c$, where $a \neq a(t)$. The significant deviations from the condition $a \neq a(t)$ for real bodies can be explained by some temperature dependence of body structures, as well as the number of shapes of carriers participating in the energy transfer. For the plate considered the specific thermal flux Q along the axis remains unchanged, i.e.

$$Q = -\lambda\frac{\partial t}{\partial x} = -ac\rho\frac{\partial t}{\partial x} = -a\frac{\partial U}{\partial t}\frac{\partial t}{\partial x} = -a\frac{dU}{dx} = \text{const}. \tag{4}$$

It follows from Eq. (4) that the specific internal energy U, and consequently also the mean energy of body particles E, depends linearly on the coordinate x.

A particle with coordinate x from particle of an isothermal layer of thickness $d\eta$, removed by distance η, acquires energy $dQ = \frac{F}{2} d\eta q(x+\eta)\int_0^{\infty} G(\sqrt{\eta^2+\zeta^2})d\zeta$. In that case the energy balance equation for layer particles with coordinate x can be represented, with account of condition (3), in the form

$$\frac{F}{2}\int_0^{\infty}\left\{\left[q(x+\eta)+q(x-\eta)-2q(x)\right]d\eta\int_0^{\infty}G(\sqrt{\eta^2+\zeta^2})\,d\zeta\right\} = 0.$$

This equation will be satisfied under the condition that $q(x+\eta)+q(x-\eta)-2q(x) = 0$. It hence follows that q, as well as E, depend linearly on the coordinate x, and, consequently, q depends linearly on E. Since particles occurring at vanishing energy level cannot emit energy, then

$$q = \varepsilon E, \tag{5}$$

where ε is the energy emission coefficient, which is characteristic of body particles and is independent of E. If energy transport in the body is realized by carriers of various shapes, then each atom, according to its number of degrees of freedom, can emit and absorb only three shapes of carriers at each moment of time. In this case relationship (5) is valid for each degree of freedom, and the total energy of a particle consists of the energies transmitted by each degree of freedom.

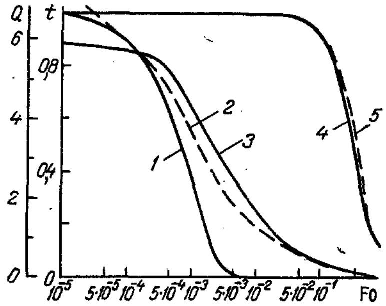

Fig. 1. Comparison of the solutions of the Fourier heat conduction equation and the integrodifferential transfer equation. Q, 10^6 W/m.

If the energy carriers are photons, then from (5) follows the following law of intensive spectral emission: atoms in the i-th energy level at frequency ν emit per unit time in a coordinate system attached to them energy quanta $h\nu$, whose magnitude is proportional to the energy level i and to the number of atoms $n_{i\nu}$, i.e.

$$q_{i\nu} = \varepsilon_\nu n_{i\nu} i h\nu. \tag{6}$$

Starting from Eq. (6), one can obtain the Planck equation for the radiative capability of an absolutely black body, the well-known particle distribution functions in energy, as well as new distribution functions for a finite number of particles and a finite upper limit in the spectrum of energy states [1]. It must be noted that in this case the necessity **is removed** of using the fundamental assumption of statistical mechanics of equal probability of all admissible states [2], whose physical essence is not totally disclosed. According to (5) one constructs the interatomic interaction potential, and on its basis one obtains an equation of state of condensed bodies, from which follow the Hooke, Gruneisen, and thermal expansion laws [3].

Substituting relations (2) and (5) into Eq. (1) and transforming from summation to integration, for a body with sufficiently large volume we reach the following integrodifferential transport equation in a Cartesian coordinate system:

$$\frac{\partial U(x,\ y,\ z,\ \tau)}{\partial x} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{s=1}^{S}\sum_{b=1}^{B_s} F_b \Big[u_{bs}\Big(x+\eta_x,\ y+\eta_y,\ z+\eta_z,\ \tau-\frac{\eta}{v_b}\Big) - u_{bs}(x,\ y,\ z,\ \tau)\Big]\frac{G_b}{4\pi\eta^2}\, d\eta_x d\eta_y d\eta_z + W. \tag{7}$$

Here $\eta = \sqrt{\eta_x^2+\eta_y^2+\eta_z^2}$; $U = \sum_{s=1}^{S}\sum_{b=1}^{B_s} n_{bs}e_{bs}$; $W = \sum_{s=1}^{S}\sum_{b=1}^{B_s} n_{bs}w_{bs}$; $u_{bs} = n_{bs}e_{bs}\varepsilon_{bs}$; $G_b = \exp\Big(-\int_0^{\eta} F_b d\eta\Big)$; B_s is the number of shapes of particles emitted by atoms of kind s, n_{bs} is the particle density of kind s, emitting carriers of shape b, and e_{bs} is the energy belonging to a single degree of freedom of a particle of kind s, emitting carriers of shape b. Introducing the averaged values of the characteristics ε, F, v, G, the integrodifferential transport equation can be written in the form

$$\frac{\partial U(r,\ \tau)}{\partial \tau} = \int_V \Big[U\Big(r+\eta,\ \tau-\frac{\eta}{v}\Big) - U(r,\ \tau)\Big] N dV + W, \tag{8}$$

where N is a function of the radius-vector r of a given point, of the radius-vector η joining the given point with an arbitrary point of the body, of microscopic characteristics, and of the body structure, $N = \varepsilon FG/(4\pi\eta^2)$. Related to the fast decrease of the function N with increasing η, the volume V over which the integration is carried out can be restricted by a sphere of radius R, determined by the condition $\Big(\int_0^{\infty} N d\eta - \int_0^{R} N d\eta\Big) \Big/ \int_0^{\infty} N d\eta \ll 1$. Expanding the

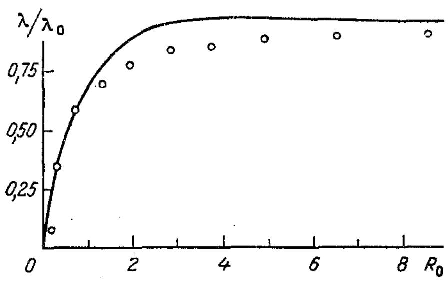

Fig. 2. Relative thermal conductivity as a function of the bar and film thickness. R_0, 10^{-7} m.

function $u_{bs}\left(x+\eta_x,\ y+\eta_y,\ z+\eta_z,\ \tau-\frac{\eta}{v}\right)$ near the point (x, y, z, τ) in a Taylor series in powers of η_x, η_y, η_z, η/v, and retaining terms up to third order of smallness, we reach a hyperbolic equation which is ordinarily used to describe intensive transport processes. At $v \to \infty$ this equation transforms to the Fourier thermal conductivity equation with a thermal conductivity coefficient for a homogeneous body

$$\lambda = \frac{1}{3} \sum_{s=1}^{S} \sum_{b=1}^{B_s} \frac{n_{bs} c_{bs} \varepsilon_{bs}}{F_b^2}. \tag{9}$$

Expression (9) agrees with experimental data and with the experimental Debye equation [1].

To find the function U at a boundary point P and its neighborhood it is necessary to determine the composition of the source function W, related to energy transfer to body particles, transferred directly from external heat sources. The simulation of various boundary conditions, particularly heat transfer conditions of the first, second, and third kind, can be carried out by conditional broadening of the emission region by attaching to the surface boundary a shell of radius R. The physical characteristics and the heat content distribution U over the shell thickness satisfy the symmetry condition with respect to the surface boundary. If near the boundary point P there are no energy sources, in this case the conditions of thermal isolation of the body are realized at the point P.

Under conditions of heat exchange of the second kind the thermal flux $Q_B(P)$ through the surface boundary is given. If the distribution of supply flux Q_B of carriers over directions is that of uniform likelihood, for points removed from the surface boundary by distance η the function is

$$W(\eta,\ \tau) = \frac{1}{2} F(\eta)\,\eta \int_0^{\infty} Q_B\left(P,\ \tau - \frac{\sqrt{\eta^2+\zeta^2}}{v}\right) \frac{G(\sqrt{\eta^2+\zeta^2})}{(\eta^2+\zeta^2)^{3/2}}\, \zeta d\zeta. \tag{10}$$

To satisfy boundary conditions of the first kind $t(P,\ \tau) = \varphi(\tau)$ the function Q_B is found as a result of simultaneous solution of the equations $\frac{\partial U}{\partial \tau} = c\rho \frac{d\varphi}{d\tau}$ and (10), as well as Eq. (8), written for the point P. Under conditions of heat exchange of the third kind $\lambda \frac{\partial t(P,\ \tau)}{\partial x} = \alpha_c [t(P,\ \tau) - t_c]$, where α_c is the heat transfer coefficient, and t_c is the medium temperature, the thermal flux $Q_B(P,\ \tau)$ is found by simultaneous solution of equations $Q_B = \alpha_c \left[\frac{U(P,\ \tau)}{c\rho} - t_c\right]$ and (8), (10).

Since the time R/v, during which the energy carrier covers distance R, is usually short in comparison with the time scale of a real thermal process, in the expansion of the function U in a Taylor series in the small time parameter R/v one may retain only the first two terms. Equation (8) then acquires the following form in Cartesian coordinates

$$\frac{\partial U(x, y, z, \tau)}{\partial \tau} = \int_{-R}^{R}\int_{-R}^{R}\int_{-R}^{R}\Big[U(x+\eta_x, y+\eta_y, z+\eta_z, \tau) - U(x, y, z, \tau) - \frac{\eta}{v}\frac{\partial U(x+\eta_x, y+\eta_y, z+\eta_z, \tau)}{\partial \tau}\Big] N(\eta)\, d\eta_x d\eta_y d\eta_z + W, \tag{11}$$

where x, y, z are the projections of the vector r on the coordinate axes, and η_x, η_y, η_z are the projections of the vector η on the x, y, z axes.

By its shape Eq. (11) is close to the integrodifferential equations encountered in the theory of radiative transfer and in studies of energy transfer by electrons in a plasma. Group methods [4-6] are widely used to solve them. A related method can also be used to solve Eq. (11). In the region considered one introduces the difference grid $x_i = i\Delta_x$, i = 0, 1, ...; $y_j = j\Delta_y$, j = 0, 1, ...; $z_m = m\Delta_z$, m = 0, 1, ...; $\tau_k = k\Delta\tau$, k = 0, 1, ... On this grid Eq. (11) is approximated by the following difference equation:

$$\frac{U_{ijm}^{k+1} - U_{ijm}^{k}}{\Delta\tau} = \sum_{\bar{i}=i-I}^{i+I}\sum_{\bar{j}=j-J}^{j+J}\sum_{\bar{m}=m-M}^{m+M}\left(U_{\bar{i}\bar{j}\bar{m}}^{k} - U_{ijm}^{k} - \frac{\eta_{\bar{i}\bar{j}\bar{m}}}{v}\frac{U_{\bar{i}\bar{j}\bar{m}}^{k} - U_{ijm}^{k-1}}{\Delta\tau}\right) N_{\bar{i}\bar{j}\bar{m}}\Delta_x\Delta_y\Delta_z + W_{ijm}^{k}. \tag{12}$$

Here $\eta_{\bar{i}\bar{j}\bar{m}} = [(\bar{i}-i)^2\Delta_x^2 + (\bar{j}-j)^2\Delta_y^2 + (\bar{m}-m)^2\Delta_z^2]^{0,5}$. The numbers I, J, and M satisfy the condition $\eta_{\bar{i}\bar{j}\bar{m}} \leqslant R$. For simplicity of calculations one can put $I\Delta_x = J\Delta_y = M\Delta_z = R$.

If the specific internal energy U = cρt and the body characteristics depend on one spatial coordinate, Eq. (8) is reduced to the form

$$\frac{\partial U(x, \tau)}{\partial \tau} = \int_0^R [U(x+\eta, \tau) + U(x-\eta, \tau) - 2U(x, \tau)] N_1 d\eta - \int_0^R \left[\frac{\partial U(x+\eta, \tau)}{\partial \tau} + \frac{\partial U(x-\eta, \tau)}{\partial \tau}\right] N_2 d\eta + W, \tag{13}$$

where

$$N_1 = \frac{1}{2} F\varepsilon \int_0^\infty \frac{\zeta d\zeta}{\zeta^2 + \eta^2} G(\sqrt{\zeta^2 + \eta^2})\, d\zeta;$$

$$N_2 = \frac{1}{2v} F\varepsilon \int_0^\infty \frac{\zeta d\zeta}{\sqrt{\zeta^2 + \eta^2}} G(\sqrt{\zeta^2 + \eta^2})\, d\zeta.$$

For a homogeneous body $N_1 = -\frac{1}{2}\varepsilon F\,\mathrm{Ei}(-F\eta)$, $N_2 = \frac{\varepsilon}{2v}\exp(-F\eta)$. According to (9) the thermal conductivity coefficient is expressed in terms of the averaged values of the parameters ε and F by the relation $\lambda = c\rho\varepsilon/3F^2$.

The integration in Eq. (13) can be carried out on the difference grid $\tau_k = k\Delta\tau$, k = 0, 1, ..., Δτ > 0; $x_i = i\Delta_x$, i = -I, -I + 1, ..., I_L + I, $\Delta_x = L/I_L$ by the following explicit difference scheme:

$$\frac{U_i^{k+1} - U_i^k}{\Delta\tau} = \sum_{j=i-I}^{i+I} Q_j^k + W_i^k. \tag{14}$$

Here $I = \left\{\frac{R}{\Delta x}\right\} + 1$, where the curved brackets imply the integral part in them, and Q_j is the energy flux acquired by a particle located in the x_i plane from the particle band $x_j < x < x_{j+1}$. If the plate consists of two layers with different characteristics σ, ε, n, F, c, ρ, while the contact boundary lies in the plane $x_C = i_C\Delta_x$, then under the condition $x_i < i_C \leq x_j$ the quantity Q_j is determined by the relation

$$Q_j^k = -\frac{1}{2}\varepsilon_2 n_2 \sigma_1 \left\{ \mathrm{Ei}(-\zeta_{i+1}) \left(\zeta_{j+1}\frac{\beta_0}{F_2} + \frac{\zeta_{j+1}^2}{2F_2^2}\delta_x U_j^k - \beta_1 \right) - \right.$$

$$- \mathrm{Ei}(-\zeta_j) \left(\zeta_j \frac{\beta_0}{F_2} + \frac{\zeta_j^2}{2F_2^2}\delta_x U_j^k - \beta_1 \right) + \tag{15}$$

$$+ \exp(-\zeta_{i+1}) \left[\beta_0 + \frac{\zeta_{j+1}+1}{2F_2^2}\delta_x U_j^k - \beta_2 + (\zeta_{j+1}+1)\beta_3 \right] -$$

$$\left. - \exp(-\zeta_j) \left[\beta_0 + \frac{\zeta_j + 1}{2F_2^2}\delta_x U_j^k - \beta_2 + (\zeta_j + 1)\beta_3 \right] \right\},$$

where

$$\zeta_{j+1} = \beta + \eta_{j+1}F_2;\ \ \zeta_j = \beta + \eta_j F_2;\ \ \beta = |x_C - x_j| \times (F_1 - F_2);$$

$$\beta_0 = U_j^k - (x_j + \beta)\,\delta_x U^k;\ \ \beta_1 = \frac{\beta}{F_2}(\delta_\tau U_j^{k-1} - x_j \delta_{x\tau} U_j^{k-1}) -$$

$$-\frac{\beta^2}{F_2^2}\delta_{x\tau} U_j^{k-1};\ \ \delta_{x\tau} U_j^{k-1} = -\frac{1}{\Delta x}(\delta_\tau U_{j+1}^{k-1} - \delta_\tau U_j^{k-1});$$

$$\beta_2 = \frac{1}{F_2}(\delta_\tau U_j^{k-1} - x_j \delta_{x\tau} U_j^{k-1}) - \frac{2\beta}{F_2^2}\delta_{x\tau} U_j^{k-1};\ \ \beta_3 = \frac{1}{F_2^2}\delta_{x\tau} U_j^{k-1};$$

$$\eta_j = \min(|x_i - x_j|,\ |x_i - x_{j+1}|);\ \ \eta_{j+1} = \max(|x_i - x_j|,\ |x_i - x_{j+1}|).$$

Here the subscripts 1 and 2 refer to quantities belonging to the first ($0 < x < x_C$) and second ($x_C < x < L$) layer.

If $\eta_{j+1} < \eta_C$, i.e., the band $x_j < x < x_{j+1}$ and a particle with coordinate x_i are within the limits of a single layer $0 < x < x_C$, then Q_j^k is determined by expression (15), in which one must put $F_2 = F_1$. For the cases when $x_j \leqslant x_C \leqslant x_i$ or $x_j \geqslant x_C$ and $x_i > x_C$, expressions for Q_j^k can be obtained from (15) by replacing the subscripts 1 and 2 in the quantities ε, σ, n, F by the opposite ones. If there are no internal energy sources in the body under consideration, and the nodal point x_i is in the neighborhood of the surface boundary x_B ($x_B = 0$ or $x_B = L$) at distance $\eta_i = |x_B - x_i| \leqslant R$, then the specific energy W_i^k imparted to the layer particles x_i by external energy sources of power Q_B is

$$W_i^k = \frac{1}{2}F\eta_i \left\{ Q_B^k[\exp(-F\eta_i) + F\eta_i\,\mathrm{Ei}(-F\eta_i)] + \frac{\eta_i}{v}\,\frac{Q_B^k - Q_B^{k-1}}{\Delta\tau}\,\mathrm{Ei}(-F\eta_i) \right\}. \tag{16}$$

For $\eta_i > R$ one can put $W_i^k = 0$. At the nodes of the supplemental layer ($i = -I, -I+1, \ldots, 1$ and $i = I_L + 1, I_L + 2, \ldots, I_L + 1$) the values of the grid function are determined after finding it at the nodes inside the layer by the relationship $U_{i_B-1}^{k+1} = U_{i_B+1}^{k+1}$.

At the node $x = x_C$, located on the contact boundary of plate layers, the thermal contents $U(x_C - 0, \tau_{k+1})$ and $U(x_C + 0, \tau_{k+1})$ are determined separately by Eq. (14). The presence of a temperature jump on the contact boundary of two bodies during some initial time segment follows from the energy balance equation in the form (14), and is due to the thermal inertia of particles. The resulting energy flux Q_C through the surface x_C is determined by algebraic summation of the energy fluxes, transported by the carriers which are emitted by the elementary layer particles $x_j < x < x_{j+1}$, $i = i_C - I, i_C - I + 1, \ldots, i_C + I - 1$.

Figure 1 shows the results of temperature field calculations by the Fourier heat conduction equation and by the integrodifferential transfer equation in a system of two unbounded ice plates ($\lambda = 2.25$; $\rho = 913$; $c = 1830$; $\varepsilon = 0.5 \cdot 10^8$) of thickness $L = 10^{-4}$ m, which at the moment of contact generation have the different temperatures t' = 1 and t" = 1. The external plate boundaries are thermally isolated.

According to the Fourier equation, at the moment $\tau = 0$, when plate contact is generated, the thermal flux is $Q_F = \infty$. In what follows the variation of the function Q_F is illustrated by curve 2 for Fourier number $\mathrm{Fo} = \tau\lambda/(c\rho L^2) > 0$. For $\tau = \mathrm{Fo} = 0$ the temperature at the contact boundary acquires instantaneously the value $0.5(t' + t'') = 0$, and remains unchanged in what follows. At the external boundary of the first plate the temperature variation t'_{BF} is characterized by curve 5.

According to the integrodifferential transport equation thermal flux Q_I (curve 3) through the contact boundary remains bounded during the whole time of contact, while the functions Q_F and Q_I practically coincide for Fo > 0.05. At the contact boundary the temperature t_C' of the first plate (curve 1) varies monotonically from 1 to 0 when Fo increases from 0 to ∞. In this case the temperature drop at the plate contact boundary, consisting of $2t'_C$, is due to the energy difference of their particles. The temperature at the external plate boundary t'_{BI} (curve 4) practically coincides with t'_{BF}.

The integrodifferential transport equation makes it possible to explain the effect of reducing the specific thermal flux through the cross section of the body with the decrease in this cross section area. Consider the heat transfer along the axis of the bar. We introduce cylindrical coordinate system (r, z, ω), in which the z-axis coincides with the axis of the bar. It is assumed that the temperature gradient is parallel to the z-axis, and that there is no thermal flux through the outer surface of the bar. An energy flux

$$dQ = df n\varepsilon U(z+\eta_z)\,\eta_r d\eta_r d\eta_z d\eta_\omega \frac{\exp\left(-F\sqrt{\eta_z^2+\eta_r^2}\right)}{4\pi(\eta_z^2+\eta_r^2)^{3/2}}$$

is incident from the elementary volume $\eta_r d\eta_r d\eta_z d\eta_\omega$, whose position is determined by the coordinates $z + \eta_z$, $r + \eta_r$, $\omega + \eta_\omega$, on the area df, located at the z cross section near the point (z, r, ω). The integration dQ over the coordinate $d\eta_r$ is carried out within the limits from 0 to R*, where for a bar of radius R_0 R* is determined from the system of equations

$$R^*\sin\omega = R_0\sin\omega_0,\ r = R^*\cos\omega - R_0\cos\omega_0,$$

where the angle ω_0 must be considered as a computational parameter.

Integration of the expression for dQ over η_r, η_z and η_ω makes it possible to calculate the distribution of the heat conduction coefficient λ over the bar cross section according to the relation $\frac{dQ_p}{df} = -\lambda\frac{\partial t}{\partial z}$, where dQ_p is the resulting energy flux through df. A noticeable deviation of the mean bar cross section value λ from the quantity $\lambda_0 = c\rho\varepsilon/3F^2$, corresponding to the case of a massive body, occurs when the bar radius R_0 is commensurate with or smaller than the quantity R. For a sufficiently thin bar one can neglect, within first approximation, the variation in the heat conduction coefficient with its cross section. In this case the integration over dQ is simplified substantially.

Figure 2 shows the results of calculating the relative thermal conductivity λ/λ_0 along the axis of a thin silver rod, found in a stationary nonequilibrium state. The points represent experimental data on the thermal conductivity of thin silver plates [7]. It is seen from Fig. 2 that the computational results are in qualitative agreement with experimental data.

The results of numerical experiments indicate that the use of the Fourier heat conduction equation to determine the temperature and heat fluxes near body contact for relatively short times of their thermal interaction (Fo < 0.05), as well as to find the thermal flux along the axis of a thin rod, gives highly inaccurate results. In this case the study of the thermal state of the system must be carried out on the basis of the integrodifferential transport equation.

NOTATION

E_α, energy of particle α relative to zero level; $\sigma_{\alpha\gamma}$, effective cross section of particle absorption α relative to carriers emitted by particles with ordinal number γ; q_α, energy of carriers emitted by a particle α per unit time; $J_{\gamma\alpha}$, specific energy flux of carriers emitted by a body particle γ incident on particle α at moment τ; Γ, number of body particles emitted by carriers reaching particle α; w_α, power supplied to particle α by internal and external energy sources relative to the body considered; W, specific power supplied to a unit volume of the body from external and internal energy sources; U, specific internal energy of the body; V, volume of the body; v, carrier propagation velocity; S, number of species of body particles; λ and a, thermal conductivity and thermal diffusivity; c, specific heat; ρ, body density; F, total effective cross section of particle absorption of unit volume; ε, energy emission coefficient; ε_ν, emission coefficient of photons of frequency ν by body particles; $n_{i\nu}$, density of particles found at frequency ν at the i-th energy level; h, Planck's constant; Fo, Fourier number; and Ei, integral exponential function.

LITERATURE CITED

1. N. I. Nikitenko, Theory of Heat and Mass Transfer [in Russian], Kiev (1983).
2. C. Kittel, Elementary Statistical Physics, Wiley, New York (1958).
3. N. I. Nikitenko, Zh. Fiz. Khim., 52, No. 4, 866-870 (1978).
4. S. Chandrasekhar, Radiative Transfer, Dover, New York (1960).
5. G. C. Pomraning, The Equations of Radiation Hydrodynamics, Pergamon Press, Oxford (1973).
6. D. Michalas and B. W. Michalas, Foundations of Radiation Hydrodynamics, New York (1984).
7. V. M. Amrosimov, B. I. Egorov, N. S. Lidorenko, and N. B. Rubashov, Fiz. Tverd. Tela (Leningrad), 2, No. 2, 147-149 (1969).

TEMPERATURE FIELD IN A HALF-SPACE WITH A FOREIGN INCLUSION

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A stationary temperature field is studied in a half-space containing a heat-liberating disclike foreign inclusion of small size. Convective heat transfer with the external medium is realized through its boundary surface.

Let us consider an isotropic half-space containing a foreign cylindrical inclusion of radius R and height l at a distance d from its boundary surface, where uniformly distributed internal heat sources of intensity q_0 act. Let the body under consideration be referred to a cylindrical coordinate system. We place the origin at the center of the inclusion. Convective heat transfer with the external medium of temperature t_c is given at the boundary surface $z = \ell - d$.

To determine the stationary temperature field, we have the heat-conduction equation [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(r, z)\frac{\partial\Theta}{\partial r}\right]+\frac{\partial}{\partial z}\left[\lambda(r, z)\frac{\partial\Theta}{\partial z}\right]=-q_0S_-(R-r)N(z), \tag{1}$$

where $\Theta = t - t_c$; $N(z) = S_-(z+l) - S_+(z-l)$.

The boundary conditions are written in the form

$$\lambda_1\frac{\partial\Theta}{\partial z}=\alpha_z\Theta \text{ for } z=-l-d,\ \Theta=0 \text{ for } r,\ z\to\infty,$$
$$\frac{\partial\Theta}{\partial r}=0 \text{ for } r\to\infty. \tag{2}$$

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